Recent advances in finite-difference analysis for convection-diffusion problems

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Abstract

In this talk, we survey our recent works [1–5] on error analysis for singularly perturbed convection-diffusion differential equations discretized by finite-difference methods. In [4], we use a newly developed preconditioning-based approach that enables us to analyze and prove uniform convergence, on Shishkin-type meshes, for a hybrid almost-third-order scheme. A Shishkin-type mesh is also used in [5], in a method based on an enhanced Kellogg-Tsan solution decomposition giving $\mathcal{O}\left(\varepsilon(\ln\varepsilon)^2 N^{-1}\right)$-accuracy, where ε is the perturbation parameter and N is the number of mesh steps. We also summarize our recent truncation error and barrier-function proof technique for the upwind scheme on a simple Bakhvalov mesh, applied to 1D problems in [1], and its extensions to 2D problems [2], and a second-order discretization on a new generalization of the Bakhvalov mesh [3].

1. T.A. Nhan, R. Vulanović. Analysis of the truncation error and barrier-functions technique for a Bakhvalov-type mesh. ETNA, Vol. 51 (2019), 315–330.
2. T.A. Nhan, R. Vulanović. The Bakhvalov mesh: a complete finite-difference analysis of two-dimensional singularly perturbed convection-diffusion problems. Numer. Algor. 87 (2021), 203–221.
3. T.A. Nhan, R. Vulanović. A new generalization of the Bakhvalov mesh for convection-diffusion problems: a hybrid scheme analysis. In preparation.
4. R. Vulanović, T.A. Nhan. Robust hybrid schemes of higher order for singularly perturbed convection-diffusion problems. Appl. Math. Comput., Vol. 386 (2020), 125495.
5. R. Vulanović, T.A. Nhan. An improved Kellogg-Tsan solution decomposition in numerical methods for singularly perturbed convection-diffusion problems. Appl. Numer. Math., Vol. 170 (2021), 128–145.